Readington Township Public Schools

Grade 6 Math Curriculum Advanced Grade 5 Math Curriculum

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I. OVERVIEW

Readington Township Public Schools' K-5 mathematics curriculum provides students with a strong foundation in mathematics content while promoting and instilling the skills of problem solving, communication in mathematics, making mathematical connections, and reasoning. Throughout the delivery of the K-5 mathematics program, various tools and technology are employed, including manipulatives, calculators, software, apps, videos, websites, and computing devices (computers, tablets, interactive whiteboards, etc.). A strong focus of the program in on promoting high levels of mathematical thought through experiences which extend beyond traditional computation.

The Math 6 course is the required, full year course for 6th grade students working on grade level. It is also the course for the Accelerated 5th grade students placed into the course by district criteria. This course is directly aligned with the New Jersey Student Learning Standards ("NJSLS") for grade 6. Through their work in this course, students will understand and apply their knowledge in real world applications. Focus will be on the content as specified in the NJSLS, as well as the NJSLS Practice Standards. The Practice Standards focus on the development of competencies used by mathematicians in all grades and throughout life.

Students in this course will study ratios, rates and proportional reasoning. They will expand their understanding of fractions to include algorithms and uses for dividing fractions. Students will use positive and negative numbers together to describe real world situations. They will order numbers and understand absolute value. Students will begin their work in Algebra as they use variables and expressions and understand the properties of numbers. They will engage in writing equations and inequalities that represent real world situations. Students will also understand area, surface area, and volume.

II. STUDENT OUTCOMES (Linked to New Jersey Student Learning Standards for Mathematics)

Ratios and Proportional Reasoning (6.RP)

Understand ratio concepts and use ratio reasoning to solve problems.

- 1. Understand the ratio concept and use ratio language to describe a relationship between two quantities.
- 2. Understand the concept of a unit rate and use rate language in the context of a ratio relationship.
- 3. Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.
 - a. Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.
 - b. Solve unit rate problems including those involving unit pricing and constant speed.
 - c. Find a percent of a quantity as a rate per 100. Solve problems involving finding the whole, given a part and the percent.
 - d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.

Number Systems (6.NS)

Apply and extend previous understandings of multiplication and division to divide whole numbers and fractions by fractions.

1. Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions using visual fraction models and equations to represent the problem.

Compute fluently with multi-digit numbers and find common factors and multiples.

- 2. Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.
- 3. Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12.
- 4. Use the distributive property to express a sum of two whole numbers 1-100 with a common factor as a multiple of a sum of two whole numbers with no common factor.

Apply and extend previous understandings of numbers to the system of rational numbers.

- 5. Understand that positive and negative numbers are used together to describe quantities having opposite directions or values; use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.
- 6. Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes to represent points on the line and in the plane with negative number coordinates.

- a. Recognize opposite signs of numbers as locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself.
- b. Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes.
- c. Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.
- 7. Understand ordering and absolute value of rational numbers.
 - a. Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram.
 - b. Write, interpret, and explain statements of order for rational numbers in real-world contexts.
 - c. Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation.
 - d. Distinguish comparisons of absolute value from statements about order.
- 8. Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane.

Expressions and Equations (6.EE)

Apply and extend previous understandings of arithmetic to algebraic expressions.

- 1. Write and evaluate numerical expressions involving whole-number exponents.
- 2. Write, read, and evaluate expressions in which letters stand for numbers.
 - a. Write expressions that record operations with numbers and with letters standing for numbers.
 - b. Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity.
 - c. Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems.
- 3. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations).
- 4. Apply the properties of operations to generate equivalent expressions.

Reason about and solve one-variable equations and inequalities.

- 5. Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true. Use substitution to determine whether a given number in a specified set makes an equation or inequality true.
- 6. Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.
- 7. Solve real-world and mathematical problems by writing and solving equations of the form x + p = q and px = q for cases in which p, q, and x are all nonnegative rational numbers.
- 8. Write an inequality of the form x > c or x < c to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form x > c or x < c have infinitely many solutions; represent solutions of such inequalities on number line diagrams.

Represent and analyze quantitative relationships between dependent and independent variables.

9. Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation.

Geometry (6.G)

Solve real-world and mathematical problems involving area, surface area and volume.

- 1. Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes.
- 2. Find the volume of a right rectangular prism. Apply the formulas V = l w h and V = B h to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world problems.
- 3. Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate.
- 4. Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures.

Statistics and Probability (6.SP)

Develop an understanding of statistical variability.

- 1. Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers
- 2. Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape.
- 3. Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.

Summarize and describe distributions.

- 4. Display numerical data in plots on a number line, including dot plots, histograms, and box plots.
- 5. Summarize numerical data sets in relation to their context, such as by:
 - a. Reporting the number of observations.
 - b. Describing the nature of the attribute, including how it was measured and units of measurement.
 - c. Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern.
 - d. Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.

Advanced Grade 5 Math additional Fifth Grade Standards

Operations and Algebraic Thinking 5.0A

Write and interpret numerical expressions

2. Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them.

Analyze patterns and relationships.

3. Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane.

Number and Operations in Base Ten 5.NBT

Perform operations with multi-digit whole numbers and with decimals to hundredths.

6. Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

Number and Operations—Fractions 5.NF

Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

- 3. Interpret a fraction as division of the numerator by the denominator ($a/b = a \div b$). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. For example, interpret 3/4 as the result of dividing 3 by 4, noting that 3/4 multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size 3/4. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?
- 4. Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.
 - a. Interpret the product $(a/b) \times q$ as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations $a \times q \div b$. For example, use a visual fraction model to show $(2/3) \times 4 = 8/3$, and create a story context for this equation. Do the same with $(2/3) \times (4/5) = 8/15$. (In general, $(a/b) \times (c/d) = ac/bd$.)
 - b. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.
- 5. Interpret multiplication as scaling (resizing), by:
 - a. Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication.

- b. Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence $a/b = (n \times a)/(n \times b)$ to the effect of multiplying a/b by 1
- 6. Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.
- 7. Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.1
 - a. Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. For example, create a story context for $(1/3) \div 4$, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that $(1/3) \div 4 = 1/12$ because $(1/12) \times 4 = 1/3$.
 - b. Interpret division of a whole number by a unit fraction, and compute such quotients. For example, create a story context for $4 \div (1/5)$, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that $4 \div (1/5) = 20$ because $20 \times (1/5) = 4$.
 - c. Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. For example, how much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 1/3-cup servings are in 2 cups of raisins?

Measurement and Data 5.MD

Convert like measurement units within a given measurement system.

1. Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real world problems

Represent and interpret data.

2. Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Use operations on fractions for this grade to solve problems involving information presented in line plots. For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally.

Geometry 5.G

Classify two-dimensional figures into categories based on their properties.

- 3. Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles.
- 4. Classify two-dimensional figures in a hierarchy based on properties.

III. ESSENTIAL QUESTIONS AND CONTENT

Number Systems

- How do you know which operation to choose when solving a real-life problem?
- How can you use repeated factors in real-life situations?
- What does it mean to multiply and divide fractions & mixed numbers?
- How is a coordinate plane used to graph and locate points that contain negative numbers?

Ratios and Proportional Reasoning

- How is a relationship between two quantities represented?
- How are rates used to describe changes in real-life problems?
- What is the connection between ratios, fractions, and percents?
- How are lengths between the customary and metric system compared?

Expressions and Equations

- How are expressions that represent a real-life problem written and evaluated?
- Does the order in which operations are performed matter?
- How are mathematical operations used to solve an equation?
- What happens to one variable when another changes?
- How are mathematical operations used to solve an inequality?

Geometry

• How is a formula for the area of a polygon derived?

- How are the lengths of line segments in a coordinate plane found?
- How are three-dimensional figures drawn in two dimensions?
- How do you measure the surface area or volume of certain shapes?

Statistics and Probability

- How is a statistical question identified?
- What are the different ways to describe an average of a data set?
- How can intervals, tables, and graphs be used to organize data?

Additional Fifth Grade Standards for Advanced Grade 5 Math

Numbers and Operations in Base Ten

• How is division with decimals related to dividing whole numbers?

Numbers and Operations-Fractions

- Can I multiply and divide fractions just like whole numbers?
- How can I use multiplication or division of fractions to solve real-world problems?
- What strategies can be used to compare fractions?
- How can I add or subtract fractions and mixed numbers?

Operations and Algebraic Thinking

- How can expressions be written, read, and used in the real world?
- How does using parentheses in an expression change the value of the expressions?

Measurement and Data

- How do I convert among different-sized standard measurement units within a given measurement system and use them in solving real problems?
- How do I make a line plot to display a data set of measurements in fractions of a unit?

Geometry

- How do I measure two and three-dimensional shapes?
- How do I classify two-dimensional figures based on a hierarchy of properties?

IV. STRATEGIES

The curriculum will be presented through a variety of strategies, based in research on middle school learning and educational best practices. Students will be engaged in meaningful lessons and activities using guided and independent practice and cooperative learning. Students will participate in hands-on activities, use manipulatives or technology where appropriate, and participate actively in class discussions.

Teachers will encourage students to employ a number of problem solving strategies, relevant to the situations they are in. They will demonstrate evidence of understanding through modeling, verbal descriptions and oral presentations. Students may also use tools of technology where needed to better enhance their ability to complete and defend their mathematical reasoning.

V. EVALUATION

- Teacher observations & anecdotal records
- Homework assignments, classwork and exit tickets.
- Notebooks
- Student projects
- Unit tests and guizzes and check ups
- Benchmark unit assessments
- Performance based assessments
- State-based assessments
- · Class participation, discussion and sharing

VI. REQUIRED RESOURCES

Big Ideas Math 6 textbook (Ron Larson and Laurie Boswell; published by Big Ideas Learning) Associated Big Ideas Record and Practice Journal

Advanced Fifth Grade

Math Expressions Common Core Grade 5, by Dr. Karen Fuson; Published by Houghton Mifflin Harcourt

Supplemental Resources

Project Based Assignment Resources – Including, but not limited to:

- Illustrative Mathematics (<u>www.illustratviemathematics.org</u>)
- The MAP Project (www.map.mathshell.org/materials/index.php)
- Explore Learning Gizmos
- Estimation 180
- Reflex Math
- Freckle
- IXL

VII. SCOPE AND SEQUENCE

Sixth Grade

Number Systems

Numerical Expressions and Factors (12 days)

- 1. Whole Number Operations
- 2. Powers and Exponents
- 3. Prime Factorization
- 4. Greatest Common Factor
- 5. Least Common Multiples

Fractions and Decimals (18 days)

- 1. Multiplying Fractions
- 2. Dividing Fractions
- 3. Dividing Mixed Numbers
- 4. Adding and Subtracting Decimals
- 5. Multiplying Decimals
- 6. Dividing Decimals

Integers and the Coordinate Plane (18 days)

- 1. Understanding Integers
- 2. Comparing and Ordering Integers
- 3. Fractions and Decimals on the Number Line
- 4. Absolute Value
- 5. Graphing on the Coordinate Plane

Ratios and Proportional Reasoning

Ratios & Percents (25 days)

- 1. Understanding and Writing Ratios
- 2. Ratio Tables
- 3. Rates
- 4. Comparing and Graphing Ratios
- 5. Percents
- 6. Solving Percent Problems
- 7. Converting Measurement

Expressions and Equations

Algebraic Expressions and Properties (14 days)

- 1. Understanding Algebraic Expressions
- 2. Writing Expressions
- 3. Order of Operations
- 4. Properties of Addition and Multiplication
- 5. Distributive Property

Equations and Inequalities (18 days)

- 1. Writing Equations in One Variable
- 2. Solving Equations Using Addition or Subtraction
- 3. Solving Equations Using Multiplication or Division
- 4. Writing Equations in Two Variables

- 5. Writing and Graphing Inequalities
- 6. Solving Inequalities Using Addition or Subtraction
- 7. Solving Inequalities Using Multiplication or Division

Geometry

Areas of Polygons (7 days)

- 1. Areas of Parallelograms
- 2. Areas of Triangles
- 3. Areas of Trapezoids
- 4. Polygons in the Coordinate Plane

Surface Area and Volume (8 days)

- 1. Three-Dimensional Figures
- 2. Surface Area of Prisms
- 3. Surface Area of Pyramids
- 4. Volumes of Rectangular Prisms

Statistics and Probability

Statistical Measures (6 days)

- 1. Introduction to Statistics
- 2. Mean
- 3. Measures of Center
- 4. Measures of Variation
- 5. Mean Absolute Deviation

Data Displays (6 days)

- 1. Histograms
- 2. Shapes of Distributions
- 3. Box and Whisker Plots

Advanced Grade 5 Math additional Fifth Grade Standards

Advanced Grade 5 Math moves at an accelerated pace by compacting the fifth and sixth grade standards.

Operations and Algebraic Thinking 5.0A

- 1. Evaluate and Write Expressions
- 2. Analyze Patterns and Relationships

Number and Operations—Fractions 5.NF

- 1. Fractions as Division
- 2. Products of Fractions
- 3. Multiplication as Scaling
- 4. Multiply Fractions in Word Problems

Number and Operations In Base Ten 5.NBT

1. Divide Whole Numbers and Decimals

Measurement and Data 5.MD

- 1. Convert Measurement Units
- 2. Make Line Plots and Interpret Data

Geometry 5.G

- 1. Classify Two-Dimensional Figures
- 2. Properties of Two-Dimensional Figures